in d_0/D and in Reynolds number. This is also in sharp contrast with the heat-transfer results for the downstream separated region. In the latter situation, large increases in Nu/Nu_{fd} were sustained as both d_0/D and Re decreased. For instance, for $Re = 10000$, the peak value of Nu/Nu_{fd} increased from 5 to 9 as d_0/D was varied from $\frac{1}{2}$ to $\frac{1}{4}$. No corresponding variation was found to occur in the upstream separated region.

The peak value of Nu/Nu_{fd} in the upstream separated region typically occurred at about two diameters upstream of the orifice. That a maximum should occur is consistent with physical reasoning, inasmuch as the eddying flow is highly constrained in the region adjacent to the mating of the tube wall and orifice.

In conclusion, the results of this investigation suggest that heat-transfer coefficients in an upstream separated region are little different from those in a thermally developed pipe flow. For practical purposes, it appears reasonable to neglect the effect of the upstream separation in computing heat-transfer results.

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HEAT AND MASS TRANSFER IN MEDIUM WITH VARIABLE POTENTIALS

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1. INTRODUCTION

Real transfer processes usually take place in a medium whose temperature and concentration vary continuously with time. For example when extracting matter out of a porous body by diffusion, it will move continuously to the body surface and then into the surrounding medium. As a result, the extracted mass concentration will continuously increase. A similar phenomenon occurs when drying; the external air humidity is increased and its temperature reduced, while moisture is evaporated and material heated. Analogous problems arise when calculating semi-coking kinetics, diffusion–electrical processes, etc.

The peculiarity of all these problems is caused by the interrelation existing between the internal and external potentials. The mathematical model of heat and mass transfer should consider the variation of the external potentials under adequate boundary conditions. Such a generalization of the boundary conditions makes possible to some extent the application of results obtained when considering transfer phenomena in a stationary or even a moving layer.

Solutions of problems of this type in pure heat or mass transfer are given in references [1–4]. Heat transfer with only the external medium temperature changing considerably with time is discussed in reference [5].

A profound theoretical analysis of transfer processes is given in reference [6]. The present paper is based on that monograph and considers heat and mass transfer in a medium with variable potentials.

2. BASIC EQUATIONS

Internal heat and mass transfer is described by a system of differential equations which, for one-dimensional bodies, can be written in dimensionless form as

$$\frac{\partial T(X, Fo)}{\partial Fo} = \frac{\partial^2 T(X, Fo)}{\partial X^2} + \frac{\Gamma}{X}\frac{\partial T(X, Fo)}{\partial X} - \varepsilon Ko \frac{\partial \theta(X, Fo)}{\partial Fo} \quad (1)$$

$$\frac{\partial \theta(X, Fo)}{\partial Fo} = Lu\left[\frac{\partial^2 \theta(X, Fo)}{\partial X^2} + \frac{\Gamma}{X}\frac{\partial \theta(X, Fo)}{\partial X}\right] - LuPn\left[\frac{\partial^2 T(X, Fo)}{\partial X^2} + \frac{\Gamma}{X}\frac{\partial T(X, Fo)}{\partial X}\right] \quad (2)$$

When symmetry exists

$$\frac{\partial T(0, Fo)}{\partial X} = \frac{\partial \theta(0, Fo)}{\partial X} = 0$$

$$T(0, Fo) \neq \infty, \qquad \theta(0, Fo) \neq \infty \tag{3}$$

The dimensionless boundary conditions of the third kind will be as follows [6]

$$\frac{\partial T(1, Fo)}{\partial X} - Bi_q[T_c(Fo) - T(1, Fo)] + (1 - \varepsilon)\, KoLuBi_m[\theta_c(Fo) - \theta(1, Fo)] = 0 \tag{4}$$

$$-\frac{\partial \theta(1, Fo)}{\partial X} + Pn\frac{\partial T(1, Fo)}{\partial X} + Bi_m[\theta_c(Fo) - \theta(1, Fo)] = 0 \tag{5}$$

3. THE PROBLEM

The variation of the external potentials with time is a function of the difference between body medium and surface potentials

$$\frac{dT_c(Fo)}{dFo} + f_1\{[T_c(Fo) - T(1, Fo)], [\theta_c(Fo) - \theta(1, Fo)]\} = 0 \tag{6}$$

$$\frac{d\theta_c(Fo)}{dFo} + f_2\{[T_c(Fo) - T(1, Fo)], [\theta_c(Fo) - \theta(1, Fo)]\} = 0 \tag{7}$$

where f_1 and f_2 are functions representing transfer kinetics. If the linearized problem in the vicinity of some values for $[T_c(Fo) - T(1, Fo)]$, $[\theta_c(Fo) - \theta(1, Fo)]$ is considered (i.e. taking only the linear part of the Taylor expansion for f_1 and f_2) instead of (6) and (7), we obtain

$$\frac{dT_c(Fo)}{dFo} + K_{11}[T_c(Fo) - T(1, Fo)] + K_{12}[\theta_c(Fo) - \theta(1, Fo)] = 0 \tag{8}$$

$$\frac{d\theta_c(Fo)}{dFo} + K_{21}[T_c(Fo) - T(1, Fo)] + K_{22}[\theta_c(Fo) - \theta(1, Fo)] = 0 \tag{9}$$

The initial potentials of the surrounding medium are

$$T_c(0) = 1, \qquad \theta_c(0) = 1 \tag{10}$$

Further, solutions are given for

$$T(X, 0) = 0, \qquad \theta(X, 0) = 0 \tag{11}$$

These solutions can easily be generalized for the case of parabolic initial distributions of transfer potentials.

4. SOLUTION OF THE PROBLEM

The equations system of differential (1), (2) describing the heat and mass transfer is solved for the conditions (3) to (5) and (8) to (11) by the use of the Laplace transform. The final results are:

$$T(X, Fo) = \frac{1 + \dfrac{1}{\Gamma + 1}\left(\dfrac{K_{22} - K_{11}}{Bi_q} Ko + \dfrac{K_{22} - K_{12}}{Bi_m Lu}\right)}{1 + \dfrac{1}{\Gamma + 1}\left[\dfrac{K_{11} + K_{21}Ko}{Bi_q} + \dfrac{K_{22}}{Bi_m Lu} + \dfrac{K_{11}K_{22} - K_{12}K_{21}}{(\Gamma + 1)Bi_q Bi_m Lu}\right]} - \sum_{n+1}^{\infty}\sum_{i=1}^{2} C_{ni}\phi_T(\vartheta_i\mu_n X)\exp[-\mu_n^2 Fo] \tag{12}$$

$$\theta(X, Fo) = \frac{1 + \dfrac{1}{\Gamma + 1}\,\dfrac{K_{11} - K_{21}}{Bi_q}}{1 + \dfrac{1}{\Gamma + 1}\left[\dfrac{K_{11} + K_{21}Ko}{Bi_q} + \dfrac{K_{22}}{Bi_m Lu} + \dfrac{K_{11}K_{22} - K_{12}K_{21}}{(\Gamma + 1)\, Bi_q Bi_m Lu}\right]} + (1/\varepsilon Ko)\sum_{n=1}^{\infty}\sum_{i=1}^{2} C_{ni}(1 - \vartheta_i^2)(\phi_T(\vartheta_i\mu_n X)\exp[-\mu_n^2 Fo] \tag{13}$$

Here

$$\vartheta_i^2 = \tfrac{1}{2}\left\{\left(1 + KoPn + \frac{1}{Lu}\right) + (-1)^i \sqrt{\left[\left(1 + KoPn + \frac{1}{Lu}\right)^2 - \frac{4}{Lu}\right]}\right\} \tag{14}$$

$$C_{n1} = \frac{2}{\mu_n\psi_n}\left\{\left[1 - \varepsilon KoK_1 - \frac{K_{11} - K_{22} + (K_{12}/\varepsilon KoK_1) - \varepsilon KoK_1K_{21}}{K_{11} - \mu_n^2 + (K_{12}/\varepsilon KoK_1)}\right]P_{n2} + \varepsilon Ko\left[1 - \frac{K_{21}}{K_{11} - \mu_n^2}\right]Q_{n2}\right\} \tag{15}$$

$$C_{n2} = -\frac{2}{\mu_n\psi_n}\left\{\left[1 - \varepsilon KoK_1 - \frac{K_{11} - K_{22} + (K_{12}/\varepsilon KoK_1)\,\varepsilon KoK_1K_{21}}{K_{11} - \mu_n^2 + (K_{12}/\varepsilon KoK_1)}\right]P_{n1} + Ko\left[1 - \frac{K_{21}}{K_{11} - \mu_n^2}\right]Q_{n1}\right\} \quad (16)$$

$$\psi_n = \vartheta_1A_{n1}P_{n2} + \vartheta_2B_{n2}Q_{n1} - \vartheta_2A_{n2}P_{n1} - \vartheta_1B_{n1}Q_{n2} \quad (17)$$

$$A_{ni} = \left\{1 + (1 - \vartheta_i^2)\,K_1 - \frac{K_{11} - K_{22} + (K_{12}/\varepsilon KoK_1) - \varepsilon KoK_1K_{21}}{K_{11} - \mu_n^2 + (K_{12}/\varepsilon KoK_1)} + \frac{1 + \Gamma}{Bi_q\mu_n^2}\frac{(K_{11} - \mu_n^2)(K_{22} - \mu_n^2) - K_{12}K_{21}}{K_{11} - \mu_n^2 + (K_{12}/\varepsilon KoK_1)}\right.$$
$$\left. + \frac{2}{Bi_q}\left[\frac{K_{11} + K_{22} - 2\mu_n^2}{K_{11} - \mu_n^2 + (K_{12}/\varepsilon KoK_1)} - \frac{(K_{11} - \mu_n^2)(K_{22} - \mu_n^2) - K_{12}K_{21}}{[K_{11} - \mu_n^2 + (K_{12}/\varepsilon KoK_1)]^2}\right]\right\}V_\Gamma(\vartheta_i\mu_n)$$
$$+ \left\{\frac{2\mu_n}{\vartheta_i}\frac{K_{11} - K_{22} + (K_{12}/\varepsilon KoK_1) - \varepsilon KoK_1K_{21}}{[K_{11} - \mu_n^2 + (K_{12}/\varepsilon KoK_1)]^2} - \frac{\vartheta_i}{Bi_q\mu_n}\frac{(K_{11} - \mu_n^2)(K_{22} - \mu_n^2) - K_{12}K_{21}}{K_{11} - \mu_n^2 + (K_{12}/\varepsilon KoK_1)}\right\}\cdot\phi_\Gamma(\vartheta_i\mu_n) \quad (18)$$

$$B_{ni} = \left\{(1 - \vartheta_n^2) + \frac{\varepsilon KoK_{21}}{K_{11} - \mu_n^2}\right\}V_\Gamma(\vartheta_i\mu_n) - \frac{\varepsilon KoK_{21}}{(K_{11} - \mu_n^2)^2}\phi_\Gamma(\vartheta_i\mu_n) + \frac{(1 - \vartheta_i^2) + \varepsilon KoPn}{Bi_m}\left[2\left\{1 + \frac{K_{12}K_{21}}{(K_{11} - \mu_n^2)^2}\right\}\right.$$
$$\left. V_\Gamma(\vartheta_i\mu_n) + \frac{(K_{11} - \mu_n^2)(K_{22} - \mu_n^2) - K_{12}K_{21}}{\mu_n^2(K_{11} - \mu_n^2)}\{(1 + \Gamma)\,V_\Gamma(\vartheta_i\mu_n) - \vartheta_i\mu_n\phi_\Gamma(\vartheta_i\mu_n)\}\right] \quad (19)$$

$$Q_{ni} = \left\{1 + (1 - \vartheta_i^2)K_1 - \frac{K_{11} - K_{22} + (K_{12}/\varepsilon KoK_1) - \varepsilon KoK_1K_{21}}{(K_{11} - \mu_n^2 + (K_{12}/\varepsilon KoK_1)}\right\}\phi_\Gamma(\vartheta_i\mu_n)$$
$$+ \frac{(K_{11} - \mu_n^2)(K_{22} - \mu_n^2) - K_{12}K_{21}}{K_{11} - \mu_n^2 + (K_{12}/\varepsilon KoK_1)}\frac{\vartheta_i}{Bi_q\mu_n}V_\Gamma(\vartheta_i\mu_n) \quad (20)$$

$$P_{ni} = \left[(1 - \vartheta_i^2) + \frac{\varepsilon KoK_{21}}{K_{11} - \mu_n^2}\right]\phi_\Gamma(\vartheta_i\mu_n) + \frac{(1 - \vartheta_i^2) + \varepsilon KoPn}{Bi_m}\cdot\frac{(K_{11} - \mu_n^2)(K_{22} - \mu_n^2) - K_{12}K_{21}}{K_{11} - \mu_n^2}\frac{\vartheta_i}{\mu_n}V_\Gamma(\vartheta_i\mu_n) \quad (21)$$

$$K_1 = \frac{1 - \varepsilon}{\varepsilon}Lu\frac{Bi_m}{Bi_q}$$

where μ_n are roots of the characteristic equation

$$Q_{n1}P_{n2} - P_{n1}Q_{n2} = 0 \quad (22)$$

$$\phi_\Gamma(x) = 1 - \frac{x^2}{2(\Gamma + 1)} + \frac{x^4}{2\,.\,4\,.\,(\Gamma + 1)(\Gamma + 3)} - \ldots = \sum_{K=0}^{\infty}(-1)^k\frac{x^{2k}}{(2k)!!\,(\Gamma + 2k - 1)!!} \quad (23)$$

$$V_\Gamma(x) = -\frac{d\phi_\Gamma(x)}{dx} = \frac{x}{\Gamma + 1} - \frac{x^3}{2(\Gamma + 1)(\Gamma + 3)} + \ldots = \sum_{K=0}^{\infty}(-1)^k\frac{x^{2k+1}}{(2k)!!\,(\Gamma + 2k + 1)!!} \quad (24)$$

For the plate ($\Gamma = 0$), cylinder ($\Gamma = 1$) and sphere ($\Gamma = 2$) the functions $\phi_\Gamma(x)$ and $V_\Gamma(x)$ can be written as follows

$$\phi_0(x) = \cos x, \quad \phi_1(x) = J_0(x), \quad \phi_2(x) = \frac{\sin x}{x}$$

$$V_0(x) = \sin x, \quad V_1(x) = J_1(x), \quad V_2(x) = \frac{-x\cos x + \sin x}{x^2}$$

When $K_{12} = K_{22} = K_{21} = 0$ the results become identical with those given in reference [5]. When $K_{11} = 0$ also, the results equal those presented in reference [6].

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A CORRELATION FOR TOTAL BAND ABSORPTANCE OF RADIATING GASES*

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NOMENCLATURE

a,	dimensionless constant defined in equation (10);
A,	total band absorptance defined in equation (1) [cm^{-1}];
A_0,	spectroscopic constant [cm^{-1}];
$\bar{A}$,	dimensionless total band absorptance defined in equation (3);
B^2,	spectroscopic constant [(atm)$^{-n}$];
C_0^2,	spectroscopic constant [cm^2/g];
f_1, f_2,	dimensionless functions defined in equation (2);
k,	spectral absorption coefficient [cm^2/g];
L,	mean beam length [cm];
P^n,	equivalent broadening pressure [(atm)^n];
r,	geometric beam length [cm];
t,	dimensionless equivalent broadening pressure defined in equation (3);
u,	dimensionless mass path length defined in equation (3);
w,	mass path length [g/cm^2].

Greek symbols

ν,	wave number [cm^{-1}];
ϕ,	angle [deg];
Ω,	solid angle [steradian].

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INTRODUCTION

THE CALCULATION of radiant energy transfer in bounded and unbounded systems requires a description of gaseous band absorption and emission phenomena. For practical purposes, it is often convenient to use the total or integrated band absorptance, A, defined as

$$A = \int_{\Delta\nu} A(\nu)\,d\nu = \int_{\Delta\nu} (1 - e^{-k(\nu)w})\,d\nu \qquad (1)$$

where $A(\nu)$ is the spectral absorptance, $\Delta\nu$ the effective band width, $k(\nu)$ the spectral absorption coefficient and w the mass path length. The spectral absorption coefficient $k(\nu)$ is, in general, a very rapidly varying function of frequency, and hence exact integration of equation (1) over a complex vibration–rotation band is formidable.

In one of the first attempts to describe total band absorptance analytically, Schack [1] approximated the spectral absorption coefficient as a function of reordered wavelength by a straight line and by a series of straight lines. The result for the straight-line approximation is a simple expression involving two correlation constants (two-parameter model), and is not a function of pressure. Penner [2] has pointed out that this approximation is of little value since it is valid only at high pressures where substantial overlapping of rotational lines occurs. Edwards and Menard [3] have also mentioned that as the mass path length becomes large, Schack's expression approaches a constant, instead of the logarithmic asymptote indicated by experimental results [4–7].

Much of the experimental band absorptance data available to date has been correlated by using linear, square root,